Università IUAV - Compito di Analisi Mat. & Geometria 30/1/14

nome cognome matr.

Avvertenze: dati personali in STAMPATELLO. Riportate **tutti i risultati** su questo foglio.

1) Determinare le caratteristiche delle matrici completa, incompleta e risolvere il seguente sistema lineare

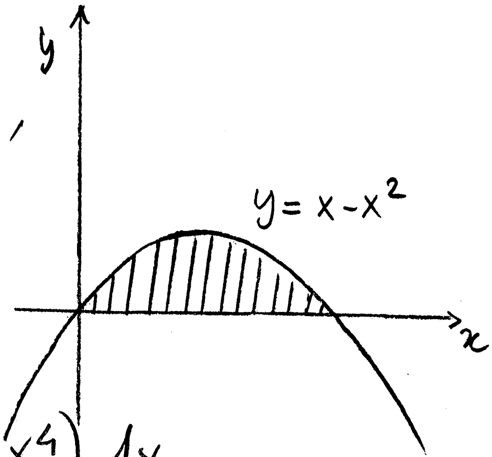
A

$$4 = 2.1 - 2 - 3 \begin{cases} x + 3y + 2w - z = 5 \\ x - y + w + 4z = 4 \\ x + 5y + 2w - 3z = 3 \\ 2y + w - 3z = 3 \end{cases}$$

2) Dati $\mathbf{u} = 3\mathbf{i} - 4\mathbf{k}$, $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, determinare i vettori $\mathbf{w}$ di norma 2 e perpendicolari a $\mathbf{u}$ e $\mathbf{v}$.

3) Determinare massimi e minimi (relativi, assoluti) della funzione $f(x) = (e^x - 1)\sqrt{8 - 2e^x}$ in $(-\infty, \ln 4]$.

4) Determinare il baricentro della parte di piano evidenziata in figura



4)

$$A = \int_0^1 (x - x^2)\, dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6},$$

$$M_{\{x=0\}} = \int_0^1 x(x - x^2)\, dx = \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12},$$

$$M_{\{y=0\}} = \frac{1}{2}\int_0^1 (x - x^2)^2 dx = \frac{1}{2}\int_0^1 (x^2 - 2x^3 + x^4)\, dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5}\right]_0^1 = \frac{1}{2}\left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right) = \frac{1}{60}$$

$$X_G = M_{\{x=0\}}/A = \frac{\frac{1}{12}}{\frac{1}{6}} = \frac{1}{2}, \quad Y_G = M_{\{y=0\}}/A = \frac{\frac{1}{60}}{\frac{1}{6}} = \frac{1}{10}.$$

2) RIDUCIAMO A SCALA LE MATRICI COMPLETA E INCOMPLETA:

$$\left(\begin{array}{cccc|c} 1 & 3 & 2 & -1 & 5 \\ 1 & -1 & 1 & 4 & 4 \\ 1 & 5 & 2 & -3 & 3 \\ 0 & 2 & 1 & -3 & 3 \end{array}\right) \xrightarrow[A_3 - A_1]{A_2 - A_1} \left(\begin{array}{cccc|c} 1 & 3 & 2 & -1 & 5 \\ 0 & -4 & -1 & 5 & -1 \\ 0 & 2 & 0 & -2 & -2 \\ 0 & 2 & 1 & -3 & 3 \end{array}\right) \xrightarrow[A_4 + \frac{1}{2}A_2]{A_3 + \frac{1}{2}A_2}$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 2 & -1 & 5 \\ 0 & -4 & -1 & 5 & -1 \\ 0 & 0 & -1/2 & 1/2 & -5/2 \\ 0 & 0 & 1/2 & -1/2 & 5/2 \end{array}\right) \xrightarrow{A_4 + A_3} \left(\begin{array}{cccc|c} 1 & 3 & 2 & -1 & 5 \\ 0 & -4 & -1 & 5 & -1 \\ 0 & 0 & -1/2 & 1/2 & -5/2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$\Rightarrow$ car(A) = car$(A|B) = 3$ $\Rightarrow$ IL SISTEMA AMMETTE $\infty^{4-3} = \infty^1$ soluzioni

RISOLVIAMO IL SISTEMA EQUIVALENTE

A

$$\begin{cases} x+3y+2w-z=5 \\ -4y-w+5z=-1 \\ -w+z=-5 \end{cases}$$

$z=t, \quad t\in\mathbb{R}$

$w=5+z=5+t,$

$$y=\frac{1-w+5z}{4}=\frac{1-5-t+5t}{4}=-1+t$$

$$x=5-3y-2w+z=5+3-3t-10-2t+t=-2-4t.$$

2) $\vec{w}\perp\vec{u},\vec{v} \iff \vec{w}\parallel\vec{u}\wedge\vec{v} \iff \vec{w}=\lambda(\vec{u}\wedge\vec{v}), \lambda\in\mathbb{R}$

$$\vec{u}\wedge\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k} \\ 3 & 0 & -4 \\ 1 & -2 & 2\end{vmatrix}=-8\vec{i}-10\vec{j}-6\vec{k}$$

POICHE' SI RICHIEDE $\|\vec{w}\|=2$, DOVRA' ESSERE

$$|\lambda|=2/\|\vec{u}\wedge\vec{v}\|=2/\sqrt{8^2+10^2+6^2}=2/\sqrt{200}=\sqrt{2}/10$$

$$\Rightarrow \vec{w}=\pm\frac{\sqrt{2}}{5}(4\vec{i}+5\vec{j}+3\vec{k}).$$

3) $f'(x)=3e^x(3-e^x)/\sqrt{8-2e^x}$ per $x<\ln 4$

QUINDI $f'(x)>0 \iff 3-e^x>0 \iff x<\ln 3$

$x<\ln 4$

$f'>0$ $\quad$ $f'<0$

$\nearrow$ $\ln 3$ $\searrow$ $\ln 4$

$x=\ln 3$ MAX. REL.

$x=\ln 4$ MIN. REL.

POICHE' $f'<0$ per $x\in(\ln 3,\ln 4)$; $f'(x)>0$ per $x<\ln 3$

$x=\ln 3$ E' ANCHE MAX. ASSOLUTO.

INFINE, TENUTO CONTO CHE $f(\ln 4)=0$

MENTRE $f(x)<0$ per $x<0$, SEGUE CHE

$x=\ln 4$ <u>NON E'</u> MIN. ASSOLUTO

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nome ______ cognome ______ matr. ______

Avvertenze: dati personali in STAMPATELLO. Riportate **tutti i risultati** su questo foglio.

1) Determinare le caratteristiche delle matrici completa, incompleta e risolvere il seguente sistema lineare

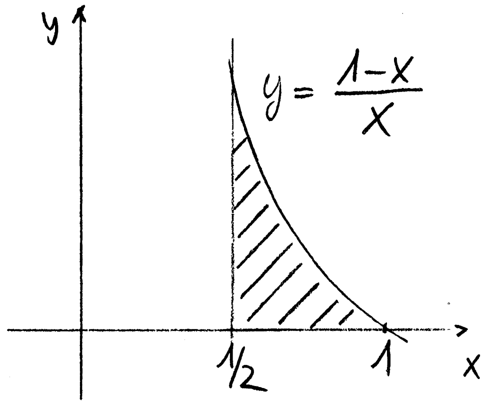
$$\begin{cases} -x+y-2w-5z=-2 \\ x-3y+4w+z=3 \\ -2y+2w-4z=1 \\ -2x-2w-14z=-3 \end{cases}$$

B

2) Dati $\mathbf{u}=\mathbf{i}-2\mathbf{k}$, $\mathbf{v}=\mathbf{i}-\mathbf{j}+2\mathbf{k}$, $\mathbf{w}=\mathbf{i}+4\mathbf{j}-\mathbf{k}$, determinare il coseno dell'angolo formato dai vettori $\mathbf{u}\wedge\mathbf{v}$ e $\mathbf{w}$.

3) Determinare massimi e minimi (relativi e/o assoluti) della seguente funzione: $f(x)=\cos x\sqrt[3]{\sin x}$ nell'intervallo $[0,\pi]$.

4) Determinare il baricentro della parte di piano evidenziata in figura.



1)
$$A=\int_{\frac12}^{1}\frac{1-x}{x}\,dx=\Big[\ln x-x\Big]_{\frac12}^{1}=\ln 2-\frac12$$

$$M_{\{x=0\}}=\int_{\frac12}^{1}x\left(\frac{1-x}{x}\right)dx=\left[x-\frac{x^2}{2}\right]_{\frac12}^{1}=\frac18$$

$$M_{\{y=0\}}=\frac12\int_{\frac12}^{1}\left(\frac{1-x}{x}\right)^2dx=\frac12\int_{\frac12}^{1}\left(\frac{1}{x^2}-\frac{2}{x}+1\right)dx$$

$$=\frac12\left[-\frac1x-2\ln x+x\right]_{\frac12}^{1}=\frac34-\ln 2$$

$$x_G=\frac{M_{\{x=0\}}}{A}=\frac{\frac18}{\ln 2-\frac12}=\frac{1}{8\ln 2-4}$$

$$y_G=\frac{M_{\{y=0\}}}{A}=\frac{\frac34-\ln 2}{\ln 2-\frac12}=\frac{3-4\ln 2}{2(2\ln 2-1)}$$

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2)
$$\left(\begin{array}{cccc|c} -1 & 1 & -2 & -5 & -2 \\ 1 & -3 & 4 & 1 & 3 \\ 0 & -2 & 2 & -4 & 1 \\ -2 & 0 & -2 & -14 & -3 \end{array}\right)\xrightarrow[A_4-2A_1]{A_2+A_1}\left(\begin{array}{cccc|c} -1 & 1 & -2 & -5 & -2 \\ 0 & -2 & 2 & -4 & 1 \\ 0 & -2 & 2 & -4 & 1 \\ 0 & -2 & 2 & -4 & 1 \end{array}\right)\xrightarrow[A_4-A_2]{A_3-A_2}$$

$$\left(\begin{array}{cccc|c} -1 & 1 & -2 & -5 & -2 \\ 0 & -2 & 2 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

car(A) = car$(A|B)=2$

$\Rightarrow$ IL SISTEMA AMMETTE $\infty^{4-2}=$
$=\infty^2$ SOLUZIONI

RISOLVIAMO IL SISTEMA EQUIVALENTE

B

$$\begin{cases} -x + y - 2w - 5z = -2 \\ -2y + 2w - 4z = 1 \end{cases} \qquad \begin{matrix} w = s \\ z = t \end{matrix} \qquad s, t \in \mathbb{R}$$

$$y = s - 2t - \frac{1}{2}$$

$$x = 2 + y - 2w - 5z = \frac{3}{2} - s - 7t.$$

3)

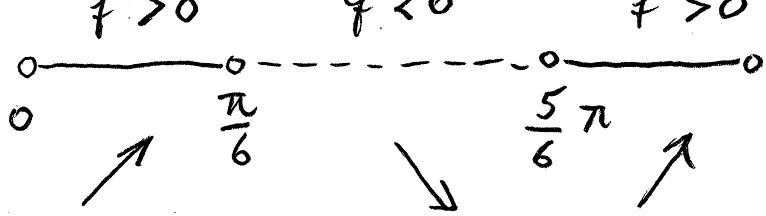
$$\vec{u} \wedge \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -2 \\ 1 & -1 & 2 \end{vmatrix} = -2\vec{i} - 4\vec{j} - \vec{k}$$

$$\cos\theta = \frac{\vec{w} \cdot (\vec{u} \wedge \vec{v})}{\|\vec{w}\| \, \|\vec{u} \wedge \vec{v}\|} = \frac{(-2) \cdot 1 + (-4) \cdot 4 + (-1)(-1)}{\sqrt{1^2+4^2+1^2}\,\sqrt{2^2+4^2+1^2}} = \frac{-17}{3\sqrt{42}}$$

4)

$$f'(x) = -\operatorname{sen} x \sqrt[3]{\operatorname{sen} x} + \cos x \, \frac{1}{3} \frac{\cos x}{\sqrt[3]{\operatorname{sen}^2 x}} = \frac{1}{3\sqrt[3]{\operatorname{sen}^2 x}} \left(1 - 4 \operatorname{sen}^2 x\right) \quad \text{per } 0 < x < \pi$$

$$\Rightarrow f'(x) > 0 \iff \begin{cases} |\operatorname{sen} x| < \frac{1}{2} \\ 0 < x < \pi \end{cases} \iff \begin{cases} 0 < x < \frac{\pi}{6} \\ \vee \\ \frac{5}{6}\pi < x < \pi \end{cases}$$



$x = \frac{\pi}{6}, \ \pi$ MAX. REL.

$x = \frac{5}{6}\pi, \ 0$ MIN. REL.

POICHE' $f(0) = f(\pi) = 0$

$$f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \frac{1}{\sqrt[3]{2}}, \quad f\left(\frac{5}{6}\pi\right) = -\frac{\sqrt{3}}{2} \frac{1}{\sqrt[3]{2}} \Rightarrow$$

$x = \frac{\pi}{6}$ MAX. ASSOLUTO, $x = \frac{5}{6}\pi$ MINI. ASSOLUTO

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1) Determinare le caratteristiche delle matrici completa, incompleta e risolvere il seguente sistema lineare

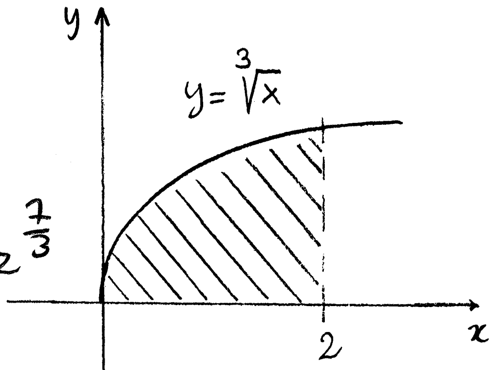
$$\begin{cases} x+3y-2w-z=1 \\ 2x-y+w+z=3 \\ x-5y+2w-3z=0 \\ -y-w-5z=-2 \end{cases}$$

C

2) Dati i vettori $\mathbf{u}=3\mathbf{i}-2\mathbf{k}$, $\mathbf{v}=\mathbf{i}-2\mathbf{j}+2\mathbf{k}$, $\mathbf{w}=\mathbf{i}+4\mathbf{j}-\mathbf{k}$, determinare λ in modo che i vettori $\mathbf{w}$ e $\mathbf{u}-\lambda\mathbf{v}$ siano ortogonali.

3) Determinare massimi e minimi (relativi e/o assoluti) della seguente funzione: $f(x)=\sin x\sqrt{\cos x}$ nell'intervallo $[-\frac{\pi}{2},\frac{\pi}{2}]$.

4) Determinare il baricentro della parte di piano evidenziata in figura.



4) $A=\int_0^2 x^{\frac{1}{3}}\,dx=\frac{3}{4}\left[x^{\frac{4}{3}}\right]_0^2=3\cdot 2^{-2/3}$

$M_{\{x=0\}}=\int_0^2 x\cdot x^{1/3}\,dx=\frac{3}{7}\left[x^{\frac{7}{3}}\right]_0^2=\frac{3}{7}2^{\frac{7}{3}}$

$M_{\{y=0\}}=\frac{1}{2}\int_0^2\left(x^{1/3}\right)^2dx=\frac{1}{2}\left[\frac{3}{5}x^{\frac{5}{3}}\right]_0^2=\frac{3}{5}\,2^{2/3}$

$X_G=\frac{M_{\{x=0\}}}{A}=\frac{\cancel{3}/7\,2^{\frac{7}{3}}}{\cancel{3}\,2^{-\frac{2}{3}}}=\frac{8}{7}\ ;\ Y_G=\frac{M_{\{y=0\}}}{A}=\frac{\cancel{3}/5\,2^{\frac{2}{3}}}{\cancel{3}\,2^{-2/3}}=\frac{2^{\frac{4}{3}}}{5}$

1) RIDUCIAMO A SCALA LE MATRICI COMPLETA E INCOMPLETA

$$\left(\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 1 \\ 2 & -1 & 1 & 1 & 3 \\ 1 & -5 & 2 & -3 & 0 \\ 0 & -1 & -1 & -5 & -2 \end{array}\right) \xrightarrow[A_3-A_1]{A_2-2A_1} \left(\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 1 \\ 0 & -7 & 5 & 3 & 1 \\ 0 & -8 & 4 & -2 & -1 \\ 0 & -1 & -1 & -5 & -2 \end{array}\right) \xrightarrow[A_3-8A_4]{A_2-7A_4}$$

$$\left(\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 1 \\ 0 & 0 & 12 & 38 & 15 \\ 0 & 0 & 12 & 38 & 15 \\ 0 & -1 & -1 & -5 & -2 \end{array}\right) \xrightarrow{A_2-A_3} \left(\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 12 & 38 & 15 \\ 0 & -1 & -1 & -5 & -2 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 1 & 3 & -2 & -1 & 1 \\ 0 & -1 & -1 & -5 & -2 \\ 0 & 0 & 12 & 38 & 15 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$\mathrm{car}(A)=\mathrm{car}(A|B)=3 \Rightarrow$ IL SISTEMA AMMETTE $\infty^{4-3}=\infty^1$ SOLUZIONI

RISOLVIAMO IL SISTEMA EQUIVALENTE C

$$\begin{cases} x+3y-2w-z=1 \\ y+w+5z=2 \\ 12w+38z=15 \end{cases} \qquad \begin{aligned} &z=t, \quad t\in\mathbb{R} \\ &w=\frac{15-38t}{12}, \\ &y=2-w-5z=\frac{9-22t}{12}, \end{aligned}$$

$$x = 1-3y+2w+z = 1-\frac{27-66t}{12}+\frac{30-76t}{12}+t=\frac{15-2t}{12}.$$

2) $\vec{u}-\lambda\vec{v}=(3-\lambda)\vec{i}+2\lambda\vec{j}-(2+2\lambda)\vec{k}$

$\vec{w}\perp\vec{u}-\lambda\vec{v} \iff \vec{w}\cdot(\vec{u}-\lambda\vec{v})=0$

$$3-\lambda+8\lambda+2+2\lambda=0$$
$$5+9\lambda=0 \qquad \Rightarrow \quad \lambda=-\frac{5}{9}.$$

3) $$f'(x)=\cos x\sqrt{\cos x}-\frac{1}{2}\frac{\sin^2 x}{\sqrt{\cos x}}=\frac{1}{2\sqrt{\cos x}}(2\cos^2 x-\sin^2 x)$$
$$=\frac{1}{2\sqrt{\cos x}}(3\cos^2 x-1) \quad \text{per} \quad -\frac{\pi}{2}<x<\frac{\pi}{2}.$$

$$f'(x)>0 \iff \begin{aligned} &\cos^2 x-\frac{1}{3}>0 \\ &-\frac{\pi}{2}<x<\frac{\pi}{2} \end{aligned} \iff \begin{aligned} &-\theta<x<\theta \\ &\theta=\arccos\left(\frac{1}{\sqrt{3}}\right) \end{aligned}$$

$f'<0$ $\qquad$ $f'>0$ $\qquad$ $f'<0$

$-\frac{\pi}{2}$ $\qquad$ $-\arccos\left(\frac{1}{\sqrt{3}}\right)$ $\qquad$ $\arccos\left(\frac{1}{\sqrt{3}}\right)$ $\qquad$ $\frac{\pi}{2}$

$\searrow$ $\qquad$ $\nearrow$ $\qquad$ $\searrow$

$$\Rightarrow \quad x_1=-\arccos\left(\frac{1}{\sqrt{3}}\right), \quad x_3=\frac{\pi}{2} \qquad \text{MIN. REL.}$$
$$x_2=\arccos\left(\frac{1}{\sqrt{3}}\right), \quad x_4=-\frac{\pi}{2} \qquad \text{MAX. REL.}$$

POICHE' $f\left(-\frac{\pi}{2}\right)=f\left(\frac{\pi}{2}\right)=0$ $\quad\Rightarrow\quad$ x_1 MIN. ASSOLUTO

$f(x_1)=-\frac{\sqrt{2}}{3^{3/4}}$, $\quad f(x_2)=\frac{\sqrt{2}}{3^{3/4}}$ $\qquad$ x_2 MAX. ASSOLUTO