Analytical Approach to Collapse Mechanisms of Circular Masonry Arch[a]

Discussion by Thomas E. Boothby[3]

From the viewpoint of the response of the arch ring, this paper is a thorough and accurate contribution to the literature on uniformly loaded masonry arches. The authors' analytical method is accurate, but limited to the loading cases that were addressed, and will only be useful for quick determination of the margins of safety for uniformly loaded arches. The problems that the authors discuss with regard to material strength and interaction between arch ring, spandrel, fill, and abutments are already solved and generally available in the British literature on the subject. Although the authors'; findings are generally in agreement with the previous literature, available experimental results seriously contradict the authors' claims concerning the contribution of spandrel walls and fill to the strength of an arch. The method proposed by the authors for the incorporation of finite compressive strength of the masonry has been previously worked out in much greater detail.

The authors present an analysis method and exact equations for the determination of thrust lines and margins of safety for semicircular arches uniformly loaded and loaded by self-weight alone. In general, it is not particularly difficult to calculate trail thrust lines for a specific geometry and specific load cases. A segmental arch, such as the one shown in Fig. 10, may be divided arbitrarily into segments (most conveniently corresponding to the voussoirs) and loaded with the self-weight of the ring segment, the overlying fill, and any superimposed load. Consider a single voussoir in a system of rectangular coordinates with the origin at the center of curvature of the intrados, as in Fig. 10. For a system subjected to gravity loads only, the thrust at joint i has components V_i and H, and is applied with eccentricity e_i, measured from the centroidal radius R. Based on the statics shown in Fig. 11, the eccentricity at joint $i + 1$ can be calculated recursively, on the basis of known quantities at joint i, as

$$e_{i+1} = [H(R + e_i)\sin \beta_i + V_i(R + e_i)\cos \beta_i - (W_{ri}x_{ri} + W_{fi}x_{fi} + W_{si}x_{si})]/[H \sin \beta_{i+1} + V_i \cos \beta_{i+1} - (W_{ri} + W_{fi} + W_{si})\cos \beta_{i+1}] - R \quad (43)$$

To begin this calculation an estimate of the variables V_1, H, and e_1 is required. These can be obtained by establishing trial values of e at each of the supports and the crown. This procedure, implemented on a spreadsheet, is easily used to verify the authors' claim that for a 17-m-span semicircular arch carrying self-weight alone, the ring thickness of 0.90 m is critical (Fig. 12). Choosing $e = 0.45$ m at the abutments and crown results in $e = -0.45$ m at the haunches, hence a five-hinge collapse mechanism. Moreover, this procedure does not require a tedious derivation of equations for each load case encountered, as required by the paper.

The authors make a number of statements on the contribution of spandrels and fill to the stability of arches: "Spandrel fill can be considered and dealt with as a vertical dead load since the horizontal action produced by it is negligible compared to its weight" and "It [the contribution of granular fill

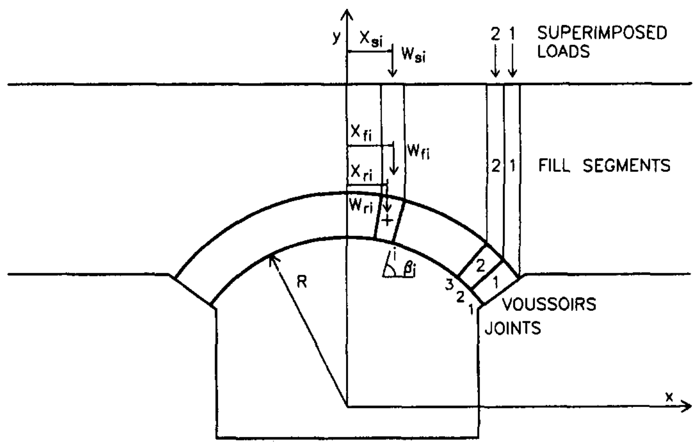

FIG. 10. Coordinate System, Joint Numbering Scheme, and Load Designations for Thrust Line Calculations

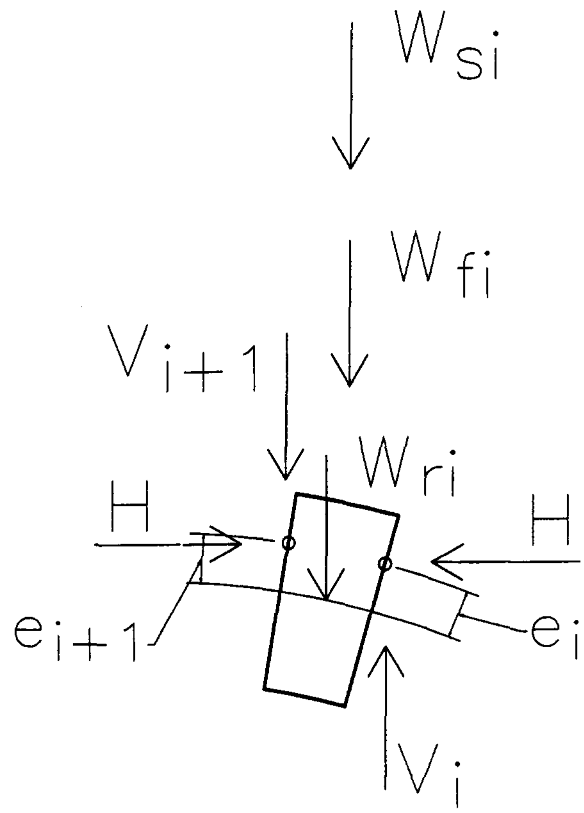

FIG. 11. Forces Acting in a Single Voussoir

to arch stability] is negligible because the internal friction angle is so great that it produces slight horizontal actions" [Blasi and Foraboschi (1994), p. 2289]. These statements are contradicted by experimental evidence from actual bridge tests (Page 1987, 1988, 1989) and tests on scale (Melbourne and Walker 1988) and full-scale models (Royles and Hendry 1991). In the full-scale model tests, models with and without fill and spandrels were tested, and the contribution of the fill and spandrels resulted in a ninefold increase in the resistance to superimposed load. When an arch with fill fails, as in Fig. 13(a), the arch ring displacements result in a tendency for the material of the fill to interpenetrate. Although this is not discussed explicitly in the paper, the authors apparently regard horizontal sliding between the fill and extrados as the mechanism to overcome this incompatibility. However, restraints at the abutments require that the center of gravity of the fill segments be raised with respect to the arch ring segments, as in Fig. 13(b), to realize the collapse mechanism. Significant work is required

[a]August 1994, Vol. 120, No. 8, by Carlo Blasi and Paolo Foraboschi (Paper 27052).

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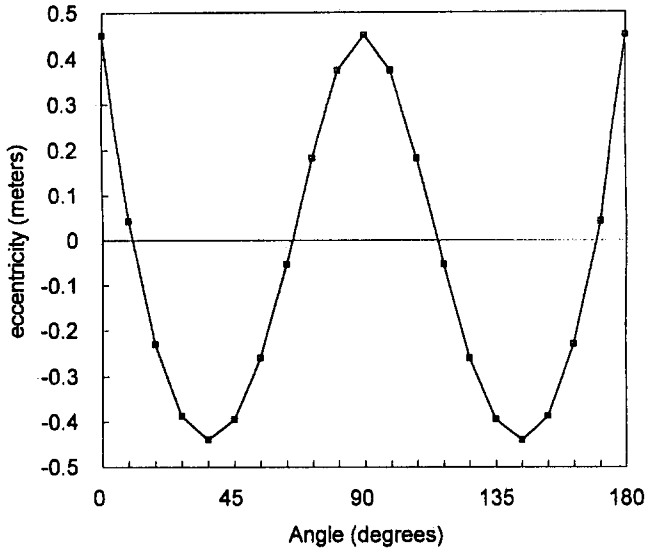

FIG. 12. Thrust Line Location for Semicircular Arch; $L = 17$ m, $s = 0.90$ m

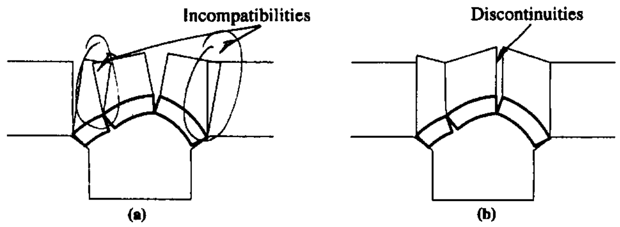

FIG. 13. (a) Fill Incompatibilities in a Four-Hinge Mechanism; (b) Fill Distortions in a Four-Hinge Mechanism

to overcome the raising of the center of gravity of the fill segments and, in most cases, this results in a considerable strengthening of the arch. This effect has now been treated analytically by Gilbert and Melbourne (1994).

The authors' discussion of the finite strength of the arch material has previously been the subject of very thorough investigations by Taylor and Mallinder (1993), Harvey (1991), and Smith et al. (1990). The most recent of these works uses actual test data on dry jointed voussoirs and develops an analytical method in the context of limit states analysis.

In summary, the authors display interesting and for the most part valid results concerning a complex and multifaceted problem. Previous developments available in the British literature have illuminated many of the problems they considered and confirm their basic approach to this problem. The authors' assumptions regarding the influence of spandrel walls and fill on the strength of arches need to be reviewed in view of experimental and analytical results available in the literature.

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Closure by Paolo Foraboschi[4] and Carlo Blasi[5]

The analytical solution of a structural problem, i.e., the description of the structural behavior through closed-form functions, should represent the primary approach to structural analysis. In fact, although numerical solutions (e.g., the trail method that is suggested to calculate thrust line) often represent the easiest and most powerful approach for predictive purpose, analytical solutions represent the most significant approach for explanation purpose (i.e., to understand the real structural behavior). Moreover, since an analytical solution consists of exact equations, all the hypotheses are clearly posed, and thus the approximations in *the solution are well known*. Thus, analytical solutions should also be researched in cases where other types of solution exist.

With reference to the masonry arch, many numerical methods existed. The oldest and most popular of these is perhaps that of Castigliano (1879, 1966). Nevertheless, our analytical approach—as the opposite to numerical approaches—explains the relationships between the geometrical characteristics of the arch and both the modes of collapse along with the load-carrying capacity. In addition, the analytical method permits explanation of how the geometrical characteristics of the arch condition retrofitting strategies (Faccio et al. 1995). Moreover, the analytical approach yields the minimum thickness to span ratio (Table 3), and systematically demonstrates that such minimum values slightly depend on the level of the uniform load.

With references to the influence of the granular fill on the strength of masonry arches, it should be observed that the most significant contribution of the fill is the horizontal thrust applied throughout the extrados of the arch. Nevertheless this contribution exists only if the reaction to this action is applied outside the structure (thrusting fill), while if the reaction acts on the abutments (no-thrusting fill) no effective contribution exists.

The masonry arch with granular fill was analytically investigated as the last step of the research. The closed-form solution was found published (Foraboschi 1995). To this objective, a soil constitutive model was defined for the fill, while the masonry was analyzed in the framework of the same no-tension constitutive law for masonry. The obtained formulation was then applied to several cases, in order to estimate the contribution of the fill to the strength of the arch (e.g., Table 4). The findings convincingly demonstrated that a thrusting fill gives significant contribution only if both its cohesion and its internal friction angle are low. Thus, if mortar is poured together with the fill, the fill behaves in practice like a dead load.

The predictions of the formulation at this last stage of development were compared with historical experimental findings (Boistard 1810), and with numerical solutions (Blasi and Foraboschi 1990). Excellent correlation resulted between an-

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TABLE 3. Minimum Thickness, S_{min}, as Function of Span, L, for Semicircular (Embracing 180°) Masonry Arch without Spandrel Fill (or with No-Thrusting Fill)

Span L (m) (1)	Minimum thickness S_{min} (m) (2)
3	0.16
4	0.23
6	0.32
8	0.44
10	0.55
12	0.65
14	0.75
16	0.86
18	0.97
20	1.08
25	1.28
30	1.59

TABLE 4. Values of Both Lower Thrust (Index min) and Related Angle of Hinge in Haunches θ_d Obtained by the Formulation Which Accounts for Fill (Apex R), and by the Formulation Which Does not Account for Fill (Apex N), for Semicircular 1-m-Depth Arch, and 19,000 N/m³ Specific Weight Fill

L (m) (1)	S (m) (2)	q (N/m²) (3)	f_r (°) (4)	C_r (N/mm²) (5)	U_r (6)	H^R_{min} (N/m) (7)	${}^{min}\theta^R_d$ (rad) (8)	H^N_{min} (N/m) (9)	${}^{min}\theta^N_d$ (rad) (10)
4	0.30	2,500	30	0.005	0	4,702	0.59	8,712	0.59
5	0.40	0	35	0.010	0.15	9,826	0.59	9,826	0.58
6	0.40	2,000	40	0	0.2	0	0.61	15,855	0.59
8	0.60	0	30	0.010	0	20,265	0.54	24,196	0.57
8	0.70	2,500	33	0	0.3	0	0.51	32,897	0.54
10	0.75	0	20	0.0175	0	32,647	0.56	35,445	0.55
10	0.70	1,500	25	0.016	0.2	37,916	0.57	40,386	0.59
10	0.80	3,000	35	0.012	0	35,875	0.56	47,870	0.55
12	0.70	0	30	0.020	0.1	46,509	0.62	46,509	0.59
12	0.80	2,500	40	0	0.05	0	0.54	58,321	0.58
14	0.80	0	30	0.020	0	59,491	0.56	62,001	0.59
14	0.80	1,500	35	0.026	0.15	65,108	0.55	68,153	0.59
14	0.90	3,000	40	0.015	0	63,401	0.62	78,552	0.59
16	0.90	0	30	0.020	0.25	49,075	0.59	79,807	0.62
16	0.90	2,000	40	0.003	0.08	76,893	0.60	89,016	0.63

Note: L = span; s = thickness, q = uniform load, f_r = friction angle of the fill, C_r = cohesion of the fill; and U_r = squeeze index of the fill (0 to 1).

alytical predictions and both experimental data and numerical solutions.

With reference to the finite strength of the arch material, the purpose of our discussion was to demonstrate that compressive strength of masonry does not influence the collapse mechanisms of the arch. To this objective, the subject that had been proposed to deal with flat arches [Blasi and Foraboschi (1989b)] was developed for masonry arches.

APPENDIX. REFERENCES

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Creep Behavior of FRP-Reinforced Wood Members[a]

Discussion by Dan A. Tingley, P.E.[3]

I have several questions with regard to the details discussed in the paper. In the section on "Material Constitutional Laws," the modulus of elasticity of wood is for bending we assume. If so, how does the fact that wood is generally considered to have a modular ratio of approximately 1.05, and that the comparison modulus of elasticity is more adversely affected by long-term loading, affect (1)–(16)? Subsequently, how then will $E_{eff}(t)$ be affected in (18)?

In the section on "Analysis of Cross Sections," shrinkage in the transverse direction will lead to reduced shear capacity of the reinforcement (RP) wood (WD) interface. Has this been considered in the model?

Further, contraction leads to strain-induced stresses. These do not seem to be considered. The research presented seems to include a full range of wet to oven-dry conditions (Tingley 1994). The shrinkage effects can lead to one-third the allowable clear wood strains. These strains can be derived by using a standard rotation matrix from the longitudinal transverse plane to the vertical longitudinal plane. Has the researcher considered this?

In the section titled "Parametric Studies," it appears that the AFRP used is low modulus (12×10^6 psi). Is this correct? If so, then it is commonly known that low modulus aramid has very poor long-term load resistance. However, high modulus aramid (18×10^6 psi) is low modulus aramid that has been stretched. This product has much better creep characteristics than E-glass.

GFRP, CFRP, and AFRP all exhibit creep strain characteristics, in tension for example, that relate to the level of long-term stress applied in the test as a percentage of ultimate tensile stress (UTS). This does not seem to have been considered in the paper. This must be considered since GFRP has a creep to failure value as a percentage of UTS one-half that of CFRP or one-half that of high modulus AFRP. Moisture effects on the creep response of FRPs does not seem to have been considered. This is also important since wood in a service dry state of 10–14% (OD basis) will saturate GFRP when it is adhered to its surface. Saturated GFRP has one-fourth the creep resistance that CFRP and AFRP (high modulus aramid) have.

Has the relationship of overstrain of reinforced wood composite to filament debond been considered?

We strongly agree with the GFRP degradation characteristics under load. Microchecking and tension-tension fatigue (particularly in high moisture–high temperature environments) lead to substantial reduction in strength characteristics.

We disagree strongly with the statement that creep behavior of FRP-reinforced wood is primarily dominated by creep in wood. The reduction of the modulus of elasticity in bending (effective) will necessarily lead to increased stress levels in the RP if plane sections remain plane and an increase in deflection occurs with constant load (creep). The RP has a much higher ultimate stress capacity in tension and its modular ratio compared to wood will mean that it tends to govern deflections when placed in the extreme tensile stress zones.

[a]February 1995, Vol. 121, No. 2, by Nikolaos Plevris and Thanasis C. Triantafillou (Paper 5979).

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